OPTIMIZATION OF BIG ECOLOGICAL SYSTEMS ON BASE OF INFORMATIONAL ENTROPY CRITERION

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Consider a macrosystem which consists M elements with stochastic behaviour. Let each element can be in any state from the class $K_1,...K_\rho$, moreover $K_1 \bigcap ... \bigcap K_\rho \equiv \varnothing$.

The sets of the states mark $Q_1...Q_\rho$, where $Q_i \in K_i\left(i \in \overline{1,\rho}\right)$. Further will be count that Q_i are discrete and containes finite number of elements.

Consider three type of elements:

1) in each state can be only one element.
2) in each state can be arbitrary number of elements.
3) average number of elements is the sets $Q_1...Q_\rho$ bigger than the capacity of their.

A stochastic behaviour of system elements generates M possible states which define the vector $N=\{N_1,...N_m\}$.

Consider the account of probable characteristic for macrostate N. Then have

$$P(N)=\prod_{n=1} P_n(N_n), \qquad \sum_{N_1,...N_m=1} P(N_1...N_m)=1 \tag{1}$$

In first case from (1) obtain

$$P_n(N_n)=\frac{G_n!}{N_n!(G_n-N_n)!} a_n^{N_n}(1-a_n)^{G_n-N_n} \tag{2}$$

By $G_n \to \infty$ and $G_n a_n \to \infty$ obtain normal approximation for binomial distribution

$$P_1(N_n)=\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left\{-\frac{(N_n-\overline{N}_n)^2}{2\sigma_n^2}\right\} \tag{3}$$

$$\langle N_n\rangle = G_n a_n, \quad \sigma_n=\sqrt{a_n(1-a_n)G_n}$$

For considered system the expression for informatical entropy has the form

$$H_1(N)=-\sum_{n=1}^{m} N_n \ln\frac{N_n}{\langle a_n\rangle}+(G_n-N_n)\ln(G_n-N_n) \tag{4}$$

where G_n is the capacity of Q_n.

For second model obtain the expressions

$$P_2(N_n)=\prod_{n=1}\frac{G_n+N_n-1}{N_n!(G_n-1)}; \; a^{N_n}(1-a_n)^{G_n-1} \tag{5}$$

$$H_2(N_n)=-\sum_{n=1}^{m} N_n \ln\frac{N_n}{a_n}-(G_n+N_n)\ln(G_n+N_n) \tag{6}$$

According to conditions of third model the expressions for the probability and the informatical entropy have the forms

$$P_3(N_n)=\prod_{n=1}^{m} e^{a_n G_n}\frac{(a_n G_n)^{N_n}}{N_n!} \tag{7}$$

$$H_3(N_n) = -\sum_{n=1}^{m} N_n \ln \frac{N_n}{a_n G_n e} \tag{8}$$

The distribution of elements connects with the consumption of different resource. In this case the set N contains all possible states. If the quantity of the resources in distribution process is finite so to exist the subset of assumption states $D \subset M$.

The condition for the existence of stationary set of macrosystem has the form of the maximum of informational entropy.

If the system to be situated in nonstationary set so in the future the system entropy increases monotonically.

The principle of maximum entropy generates the procedure of the finding of this state.

On first step we consider any abstract macrosystem in which stochastic mechanism of elements distribution on states Q_i is analogous to real system but resources are infinite. In this system all states are possible. On this set of possible states we can define the distribution function and the informational entropy.

On second step we assume that the resources are finite then in the possible states is allocated the subset of states for which the consumption is finite.

The expression for the entropy have the forms: (4), (6), (8) and the conditions of maximum entropy

$$\mathrm{H}_k(N) = \max, \quad (k = 1,2,3) \tag{9}$$

where $N \in D \subset M$.

For each system model can be considered linear and nonlinear schemes of the recourse consumption.

On base Lagranrg principle define the essential conditions of maximum entropy. In general case Lagranrg function has the form

$$L(N,\lambda) = H(N) + \sum_{k=1}^{p} \lambda_k (q_k - \varphi_k(N)) + \sum_{k=1}^{r} \lambda_k (g_k - \varphi_k(N)) \tag{10}$$

$$\varphi_k(N) = q_k, \; k \in \overline{1,p}; \quad \varphi_k(N) \le q_k, \; k \in \overline{p+1,r}$$

$$D = \{N : \varphi_k(N) = q_k, \; k \in \overline{1,p}; \; \varphi_k(N) \le q_k, \; k \in \overline{p+1,r}\}$$

Thus, we have the problem of mathematical programming for the finding of optimal state.

The conditions of the optimum for the considered models have the forms:

$$1) \left(\ln(G_n - N_n^*) - \ln \frac{N_n^*}{\langle a_n \rangle} \right) + \nabla_{N_n} \Phi(N^*, \lambda^*) = 0, \; n \in \overline{1,m} \tag{11}$$

$$2) \left(\ln(G_n - N_n^*) - \ln \frac{N_n^*}{a_n} \right) + \nabla_{N_n} \Phi(N^*, \lambda^*) = 0, \; n \in \overline{1,m} \tag{12}$$

$$3) \; -\ln \frac{N_n^*}{a_n G_n} + \nabla_{N_n} \Phi(N^*, \lambda^*) = 0, \; n \in \overline{1,m} \tag{13}$$

In case when the resource consumption is linearly the problem is simplified and the equations for first model have the form

$$H_1(N) = \max, \; \sum_{n=1}^{m} t_{kn} N_n = q_k, \; k \in \overline{1,n} \tag{12}$$

$$L_1(N,\lambda) = H_1(N) + \sum_{k=1}^{r} \lambda_k \left(q_k - \sum_{n=1}^{m} t_{kn} N_n \right) \tag{13}$$

The essential and sufficient conditions of the optimum for the problem (12), (13) have the form

$$\frac{\partial L_1}{\partial N_n} = \ln\frac{G_n - N_n}{N_n}\langle a_n\rangle - \sum_{k=1}^{r}\lambda_k t_{nk} = 0$$
$$\frac{\partial L_1}{\partial \lambda_k} = q_k - \sum_{n=1}^{m} t_{km} N_n = 0,\ k\in\overline{1,r}\quad n\in\overline{1,m} \tag{14}$$
In this case from the equation (14) follows the analytical solution
$$N_n^* = \frac{G_n}{1 + b_n \exp\left(\sum_{j=1}^{r}\lambda_j t_{jn}\right)},\ b_n = \langle a_n\rangle^{-1}\quad n\in\overline{1,m} \tag{15}$$
For second model analogous we obtain
$$H_2(N) = \max,\ \sum_{n=1}^{m} t_{kn} N_n = q_k,\ k\in\overline{1,r} \tag{16}$$
$$L_2(N) = H_2(N) + \sum_{k=1}^{r}\lambda_k\left(q_k - \sum_{n=1}^{m} t_{kn} N_n\right) \tag{17}$$
$$\frac{\partial L_2}{\partial N_n} = \ln\frac{G_n + N_n}{N_n} a_n - \sum_{k=1}^{r}\lambda_k t_{kn} = 0,\ n\in\overline{1,m}$$
$$\frac{\partial L_2}{\partial \lambda_k} = q_n - \sum_{n=1}^{m} t_{kn} N_n = 0,\ k\in\overline{1,r} \tag{18}$$
$$N_n^* = \frac{G_n}{C_n \exp\left(\sum_{j=1}^{r}\lambda_j t_{jn}\right) - 1},\ C_n = 1/a_n\quad n\in\overline{1,m} \tag{19}$$
And for third model we have
$$H_3(N) = \max,\ \sum_{n=1}^{m} t_{kn} N_n = q_k,\ k\in\overline{1,r} \tag{20}$$
$$L_3(N,\lambda) = H_3(N) + \sum_{k=1}^{r}\lambda_k\left(q_k - \sum_{n=1}^{m} t_{kn} N_n\right) \tag{21}$$
$$\frac{\partial L_3}{\partial N_n} = -\ln\frac{N_n}{a_n G_n} - \sum_{k=1}^{r}\lambda_k t_{kn} = 0,\ n\in\overline{1,m} \tag{22}$$
$$\frac{\partial L_3}{\partial \lambda_k} = q_n - \sum_{n=1}^{m} t_{kn} N_n = 0,\ k\in\overline{1,r} \tag{23}$$
$$N_n^* = a_n G_n \exp\left(-\sum_{j=1}^{r}\lambda_j t_{jn}\right),\ n\in\overline{1,m} \tag{24}$$
These solutions of optimization problem for the system with stochastic behaviour of elements can be founded in the analytical form when the system has finite linear consumption. If the consumption is nonlinear then the solution of the problem is founded by numerical methods.

References

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